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AN APPLICATION OF EXTREME VALUE THEORY FOR MEASURING FINANCIAL RISK IN THE URUGUAYAN PENSION FUND

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Abstract

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Traditional methods for financial risk measures adopt normal distributions as a pattern of the financial return behavior. Assessing the probability of rare and extreme events is an important issue in the risk management of financial portfolios. In this paper, we use Peaks Over Threshold (POT) model of Extreme Value Theory (EVT), and General Pareto Distribution (GPD), which can give a more accurate description on tail distribution of financial losses. The EVT and POT techniques provide well established statistical models for the computation of extreme risk measures like the Return Level, Value at Risk and Expected Shortfall. In this paper we apply these techniques to a series of daily losses of AFAP SURA over an 18-year period (1997-2015), AFAP SURA is the second largest pension fund in Uruguay with more than 310,000 clients, and over USD 2 billion assets under management. Our major conclusion is that the POT model can be useful for assessing the size of extreme events. VaR approaches based on the assumption of normal distribution overestimate low percentiles (due to the high variance estimation), and underestimate high percentiles (due to heavy tails). The absence of extreme values in the assumption of normal distribution underestimate the Expected Shortfall estimation for high percentiles. The extreme value approach appears consistent with respect to the actual losses observed.

Keywords: *Extreme Value Theory, General Pareto Distribution, Peaks Over Threshold, Risk Measures, Value at Risk, Pension Fund.*

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UNA APLICACIÓN DE LA TEORÍA DEL VALOR EXTREMO PARA LA MEDICIÓN DEL RIESGO FINANCIERO EN EL FONDO DE PENSIONES URUGUAYO

Resumen

Los métodos tradicionales para las medidas de riesgo financiero adoptan distribuciones normales como un patrón del comportamiento del retorno financiero. Evaluar la probabilidad de eventos raros y extremos es un tema importante en el manejo del riesgo de las carteras financieras. En este trabajo, utilizamos el modelo POT (Peaks Over Threshold) de la Teoría del Valor Extremo (EVT) y General Pareto Distribution (GPD), que puede dar una descripción más precisa de la distribución de la cola de las pérdidas financieras. Las técnicas EVT y POT proporcionan modelos estadísticos bien establecidos para el cálculo de medidas extremas de riesgo como el Nivel de Retorno, el Valor en Riesgo y la Pérdida Esperada. En este trabajo aplicamos estas técnicas a una serie de pérdidas diarias de AFAP SURA en un período de 18 años (1997-2015), AFAP SURA es el segundo mayor fondo de pensiones en Uruguay con más de 310.000 clientes y más de 2.000 millones de dólares de activos Bajo gestión. Nuestra principal conclusión es que el modelo POT puede ser útil para evaluar el tamaño de eventos extremos. Los enfoques de VaR basados en el supuesto de distribución normal sobrestiman los percentiles bajos (debido a la alta estimación de la varianza) y subestiman los percentiles altos (debido a colas pesadas). La ausencia de valores extremos en la hipótesis de distribución normal subestima la estimación de déficit esperado para percentiles altos. El enfoque de valor extremo parece coherente con respecto a las pérdidas reales observadas.

Palabras clave: *Teoría del Valor Extremo, Distribución General de Pareto, Picos sobre Umbral, Medidas de Riesgo, Valor en Riesgo, Fondo de Pensiones.*

1. Introduction

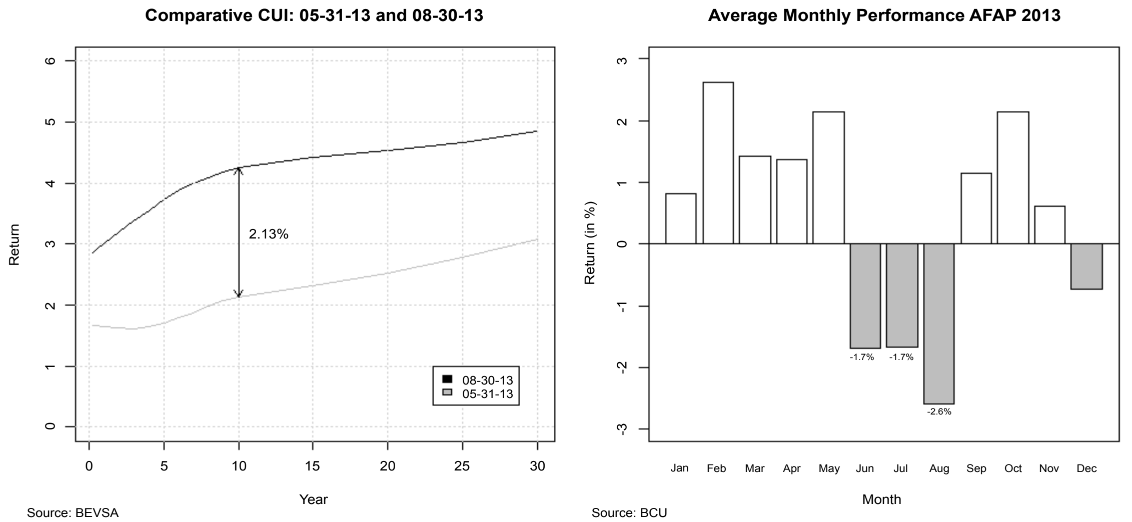
The Uruguayan pension system comprises, on the one hand, a pension scheme based on intergenerational solidarity or pay-as-you-go scheme that is administered by the Banco de Previsión Social (BPS); and, on the other hand, by an individual savings scheme which is administered by private savings firms (Administradoras de Fondos de Ahorro Provisional - AFAP). This system combines solidarity with individual savings in order to achieve financial balance in social security. A reform implemented in 1996 meant to deal with a pension crisis originated in flaws in a previous social security regime (Forteza et al. 1999).

At present, there are four AFAPs in the Uruguayan market (República AFAP, AFAP SURA, Unión Capital AFAP and Integración AFAP). According to Banco Central del Uruguay (BCU), at the close of 2015, the AFAPs managed USD 12 billion, approximately 90% of which is invested in Uruguayan assets, of which 60% is invested in sovereign assets. For this work, we use the daily NAV series of AFAP SURA over a period of eighteen years (1997-2015).

AFAP SURA has more than 310,000 clients (almost 10% of Uruguay's total population) and assets under management over USD 2 billion, being the second largest pension fund manager in Uruguay. The quota value or net asset value (NAV) is the value per share of a pension fund on a specific date. In the context of Uruguayan pension funds, NAV per share is computed once per day based on the closing market prices of the securities in the portfolio. All of the buy and sell orders for pension funds are processed at the NAV of the trade date.

The last years have been characterized by significant instabilities in financial markets. As an example of this, in mid-2013, because of a FED announcement, the yield curve of inflation-indexed bonds estimated by the Bolsa Electrónica de Valores (BEVSA) had a significant increase, corresponding to an important drop in all the bond prices. The movement in the 10-year UI yield bond was larger than 2% (see left panel of Figure 1). This situation motivated a large loss for corporate investment institutions, as pension funds, with a cost of approximately the 5% of the total portfolio (see right panel of Figure 1). This led to numerous critics about the existing risk management systems and motivated the search for more appropriate methodologies for extreme risk measures.

Figure 1: Comparative CUI and Average Monthly Performance AFAP 2013.



The purpose of this paper is to compare different methodologies to calculate risk measures for Uruguayan pension funds such as Value at Risk, Expected Shortfall and Return Level. Traditional statistical methods for financial risk measures fit models to all data even if primary focus is on extremes. It is for this reason that it is common to see in literature the normal distribution assumption for financial returns. This assumption provides a good approximation for the average of financial returns (due the central limit theorem) but does not provide a good fit for the extreme values.

The Extreme Value Theory (EVT) provides well-established statistical models for the computation of extreme risk measures. EVT became important in the 1920s with problems primarily related to hydrology and led to the first fundamental theorem of Fisher-Tippet (1928), then Gnedenko (1948). Another point of view arose in the 70s with the second fundamental theorem of Extreme Value Theory when Pickands (1975) and Balkema-de Haan (1974) characterized the asymptotic tail distribution as a Generalized Pareto Distribution (GPD) family.

EVT is a well-known technique in many fields of applied sciences including engineering and insurance (McNeil, 1999; Embrechts et al., 1999; Reiss and Thomas, 1997 and Giesecke & Goldberg, 2005). Numerous research studies surfaced recently which analyse the extremes in the financial markets due to currency crises, stock market turmoil and credit defaults. The behavior of financial series tail distributions has, among others, been discussed in Onour (2010), Gilli and Këllezi (2006), Loretan and Phillips (1994), Longin (1996), Daniels-son and de Vries (2000), Kuan and Webber (1998), Straetmans (1998), McNeil (1999), Jondeau and Rockinger (1999), Neftci (2000) and McNeil and Frey (2000).

The paper is structured as follows: first, we present the different measures of risk and then a description of the theory of extreme value. Then we will present the results of the study for the data series and end with the conclusion.

2. Risk Measures

Financial risk is the prospect for financial loss due to unforeseen changes in underlying risk factors (these factors are those that provide uncertainty in financial results). Financial risks can be classified in different ways, such as market risk, credit risk (or the risk of loss arising from the failure of a counterparty to make a promised payment), liquidity risk, operational risk (or the risk of loss arising from the failures of internal systems or the people who operate in them) and others (as legal risk, reputational risk). Market risks, in turn, can be classified as interest rate risks, equity risks, exchange rate risks, or commodity price risks (Dowd 2002).

In this section we discuss statistical summaries of the loss distribution that quantify the portfolio risk. We call these summaries as risk measures. First, we describe the risk factor, the loss distribution and returns. Then we introduce the so-called axioms of coherence, which are properties deemed desirable for measures of risk. Thereafter, we discuss two widely used measures of financial risk: Value at Risk (VaR) and Expected Shortfall and the return level (R). These risk measures consider only the downside risk, i.e. the right tail of the loss distribution.

Risk Factor, Loss Distribution and Return

Consider a portfolio of financial assets and let V_t denote its current value. The portfolio value is assumed to be observable at time t. The portfolio loss over the time interval from t to $t+1$ is written as

$$L_{t+1} = - (V_{t+1} - V_t)$$

Because V_{t+1} is unknown, L_{t+1} is random from the perspective of time t. The distribution of L_{t+1} will be referred to as the loss distribution. The portfolio value V_t will be modeled by a function of time and a set of d underlying risk factor. We write

$$V_t = f (t, Z_t)$$

For some measurable function $f\colon R+ \times R^d \rightarrow R$, where $Z_t = (Z_{t,1},.. Z_{t,d})'$ denotes a d-dimensional vector of risk factors. We define the series process of risk factor change $\{X_t\}_{teN}$, where $X_t = Z_t - Z_{t-1}$. Using the function f we can relate the risk factor changes to the changes in the portfolio value as

$$L_{t+1} = - (f (t+1, Z_t+X_{t+1}) - f (t, Z_t))$$

The portfolio loss can also take the form of arithmetic returns loss and is defined as:

$$r_t = - (V_t - V_{t-1}) / V_{t-1}$$

Which is the same as the L_t over period t divided by the value of the portfolio at the end of $t-1$. The returns loss can be interpreted as the relative loss of the portafolio. Is common in risk measures to use the return loss (r_t) instead of the portafolio losses (L_t), this is because V_t changes over the time.

Coherent Measures of Risk

Artzner et al. (1999) argue that an appropriate measure of risk should satisfy a set of properties termed as the axioms of coherence. Let financial risk be represented by a set M interpreted as portfolio losses, i.e. L in M. Risk measures are real-valued functions ρ:M→R. The amount ρ(L) represents the capital required to cover a position facing a loss L. The risk measure ρ is coherent if it satisfies the following four axioms:

- Monotonicity: $L_1 \leq L_2 \rightarrow \rho(L_1) \leq \rho(L_2)$.
- Positive homogeneity: $\rho(\lambda L) = \lambda \rho(L)$, for all $\lambda > 0$.
- Translation invariance: $\rho(L+l) = \rho(L)+l$, for all l in R.
- Subadditivity: $\rho(L_1+L_2) \leq \rho(L_1) + \rho(L_2)$

Monotonicity states that positions that lead to higher loss in every state of the world require more risk capital. Positive homogeneity implies that the capital required to cover a position is proportional to the size of that position. Translation invariance states that if a deterministic amount l is added to the position, the capital reeded to cover L is changed by precisely that amount. Subadditivity reflects the intuitive property that risk should be reduced or at least not increased by diversification, i.e. the amount of capital needed to cover two combined portfolios should not be greater than the capital needed to cover the portfolios evaluated separately.

Value at Risk

Value-at-Risk is defined as the sufficient capital to cover, in most instances, losses from a portfolio over a holding period of a fixed number of days (Gilli and Kellezi, 2006). Assume a random variable X with continuous distribution function F models losses on a certain financial portfolio over a certain time horizon. VaR_α can then be defined as the α-the quantile of the distribution F

$$VaR_\alpha = F^{-1}(1 - \alpha), \tag{1}$$

Where F^{-1} is defined as the inverse of the distribution function F. For this paper we compute a 5%, 2.5%, 1% and 0.5% *VaR* over a one-day holding period. For example, under the assumption of normal distribution, $F \sim N(\mu,\sigma)$.

However, by definition VaR_α gives no information about the size of the losses that occur with probability smaller than 1-α, i.e. the measure does not tell how bad it gets if things go wrong (Ramaswamy 2004). Given these problems with VaR_α, we seek an alternative measure which satisfies this.

Expected Shortfall

Another measure of risk is the expected shortfall *(ES)* or the tail conditional expectation that estimates the potential size of the loss exceeding *VaR* (Gilli and Kellezi, 2006). The expected shortfall is defined as the expected size of a loss that exceeds VaR_α

$$ES_\alpha = E(X \mid X > VaR_\alpha) \qquad (2)$$

Expected Shortfall, as opposed to Value at Risk, is a coherent risk measure in the sense that satisfies properties of monotonicity, sub-additivity, homogeneity, and translational invariance (Gilli and Kellezi, 2006).

Return Level

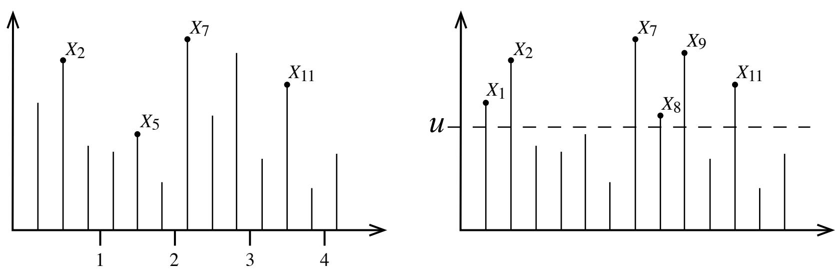
If H is the distribution of the maximum observed over successive non overlapping periods of equal length, the return level $R^m_k = H^{-1}(1- 1/m)$ is the level expected to be exceeded in one out of m periods of length k. For example, assuming a model for the annual maximum, the 15-years return level R^{15}_{365} is on average only exceeded in one year out of every 15 years. The return level can be used as a measure of the maximum loss of a portfolio, a rather more conservative measure than the Value-at-Risk (Gilli and Kellezi, 2006).

Extreme Value Theory

When modeling the maximum of a random variable, Extreme Value Theory (EVT) plays the same fundamental role as the central limit theorem when modeling sums of random variables. This is important because under certain conditions, any unknown distribution can be approximated with the Generalized Pareto Distribution. Thus, we argue that EVT provides simple parametric models to capture the extreme tails of a distribution.

There are two related ways of identifying extremes in real data. Let us consider a independent and identically distributed random variable representing daily losses. The first approach considers the maximum the variable takes in successive periods. These selected observations constitute the extreme events, also called block (or per period) maxima. In the left panel of Figure 2, the observations $\mathbf{X_2, X_5, X_7}$ and $\mathbf{X_{11}}$ represent the block maxima for four periods of three observations each.

Figure 2: Block-maxima (left panel) and excesses over a threshold *u* (right panel).

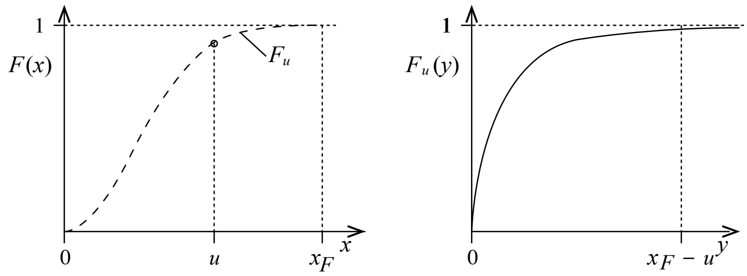


The second approach, called Peak Over Threshold (POT), focuses on the realizations exceeding a given (high) threshold u. The observations $\boldsymbol{X_1, X_2, X_7, X_8, X_9}$ and $\boldsymbol{X_{11}}$ in the right panel of Figure 2, all exceed the threshold u and constitute extreme events. Then the POT method is more efficient in terms of data usage (Embrechts 1999) and is the chosen approach for this paper.

Peak Over Threshold

The POT method considers the distribution of exceedances over a certain threshold. Our problem is illustrated in Figure 3, we consider an (unknown) distribution function F of a random variable X. We are interested in estimating the distribution function F_u, for values of a x above a certain threshold u.

Figure 3: Distribution function F and excess distribution F_u



The distribution function $\boldsymbol{F_u}$ is called the excess distribution function and is defined as

$$F_u(y) = P(X - u \mid X > u) = \frac{F(u+y) - F(u)}{1 - F(u)}, \qquad 0 \le y \le x_F - u \qquad (3)$$

Where X is a random variable, u is a given threshold, $y=x-u$ are the excesses and $x_F < \infty$ is the right endpoint of F.

The realizations of the random variable X lie mainly between 0 and u and therefore the estimation of F in this interval generally poses no problems. The estimation of the portion F_u however might be difficult as we have in general very little observations in this area.

At this point EVT can prove very helpful as it provides us with a powerful result about the excess distribution function F_u which is stated in the following theorem (Balkema and de Hann, 1974; Pickands 1975):

Theorem 1: For a large class of underlying distribution F, the excess distribution function F_u can be approximated by GPD for increasing threshold u.

$$F_u(y) \approx G_{\xi\beta}(y), u \longrightarrow \infty$$

Where $G_{\xi\beta}$ is the Generalized Pareto Distribution (GDP) which is given by

$$G_{\xi\beta}(y) = \begin{cases} \left(1 + \frac{\xi}{\beta} y\right)^{-1/\xi} & if\ \xi \neq 0 \\ 1 - e^{-y/\beta} & if\ \xi = 0 \end{cases} \qquad (4)$$

for y $\in [0,(x_F\text{-}u)]$ if $\xi < 0$ and y $\in [0,-\beta/\xi]$\$ if $\xi < 0$. Here ξ is the shape parameter and β is the scale parameter for GPD.

Thus, for any distribution F, the excess distribution F_u converges (uniformly) to a Generalized Pareto distribution (GPD) as the threshold u is raised. We define the mean excess function for the GPD with parameter $\xi < 1$ as

$$e(z) = E(X - z \mid X > z) = \frac{\beta + \xi z}{1 - \xi}, \quad \beta + \xi z > 0 \tag{5}$$

This function gives the average of the excesses of X over a varying values of the threshold

3. Dependent Sequences

The POT method is obtained through mathematical arguments that assume an underlying process consisting of a sequence of independent random variables. However, for the types of data to which extreme value models are commonly applied, temporal independence is usually an unrealistic assumption. Various suggestions, with different degrees of sophistication, have been made for dealing with the problem of dependent exceedances in the threshold exceedance model. The most widely-adopted method is declustering (Coles 2001), which corresponds to a filtering of the dependent observations to obtain a set of threshold excesses that are approximately independent. This works by:

1. Using an empirical rule to define clusters of exceedances.
2. Identifying the maximum excess within each cluster.
3. Assuming cluster maxima to be independent, with conditional excess distribution given by the GPD.
4. Fitting the GDP to the cluster maxima.

Risk Measures under Extreme Value Theory

Assuming a GPD function for the tail distribution, VaR_α, ES_α and R_m^k can be defined as a function of GPD parameters (Singh et al. 2011). For equation (3), if we denote $x=u+y$ then

$$F(x) = (1 - F(u))F_u(y) + F(u)$$

and replacing Fu by the GPD and $F(u)$ by the empiric estimate $(n-N_u)/n$, where n is the total number of observations and N_u the number of observations above the threshold u, we obtain

$$\widehat{F}(x) = \frac{N_u}{n}\left(1 - \left(1 + \frac{\widehat{\xi}}{\widehat{\beta}}(x-u)\right)^{-1/\xi}\right) + \left(1 - \frac{N_u}{n}\right) = 1 - \frac{N_u}{n}\left(1 + \frac{\widehat{\xi}}{\widehat{\beta}}(x-u)\right)^{-1/\xi} \tag{6}$$

Inverting equation (6) for a given probability α gives

$$VaR_\alpha = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}\alpha\right)^{-\xi} - 1\right) \tag{7}$$

If we add and subtract VaR_α in the equation (2) and we obtain

$$ES_\alpha = VaR_\alpha + E(X - VaR_\alpha \mid X > VaR_\alpha)$$

where the second term on the right is the expected value of the exceedances over the threshold VaR_α. Then, for equation (5) where $z = VaR_\alpha - u$ and $\xi < 1$ we have

$$ES_\alpha = VaR_\alpha + \frac{\beta + \xi(VaR_\alpha - u)}{1-\xi} = \frac{VaR_\alpha}{1-\xi} + \frac{\beta - \xi u}{1-\xi} \tag{8}$$

We know that

$$P(X > x \mid X > u) = \frac{P(X > x)}{P(X > u)} = \left[1 + \frac{\xi}{\beta}(x-u)\right]^{-1/\xi}$$

Hence, the level x_m that is exceeded on average once every m observations is the solution of

$$P(X > x) = \frac{N_u}{n}\left[1 + \frac{\xi}{\beta}(x_m - u)\right]^{-1/\xi} = \frac{1}{m}$$

where $P(X > u) = N_u / n$ is the empiric estimate. Rearranging,

$$x_m = u + \frac{\beta}{\xi}\left[\left(\frac{mN_u}{n}\right)^{\xi} - 1\right]$$

For presentation, Coles (2001) argue that it is often more convenient to give return levels on an annual scale, so that the M-year return level is the level expected to exceed once every M years. If there are k observations per year, this corresponds to the m-observation return level, where $m = M \times k$. Hence, the M-year return level is defined by

$$x_M = u + \frac{\beta}{\xi}\left[\left(\frac{kMN_u}{n}\right)^{\xi} - 1\right]$$

.

4. Empirical Results

We consider an extreme value approach working with the daily losses series of AFAP SURA NAV over a period of eighteen years (1997-2015). The empirical study uses the series of daily losses of AFAP SURA NAV, containing 4,802 trading days. The left panel of Figure 4 shows a graph of the daily evolution of AFAP SURA NAV values, and the right panel the daily return.

Figure 4: Daily evolution and return of AFAP SURA NAV since 1997 to 2015.

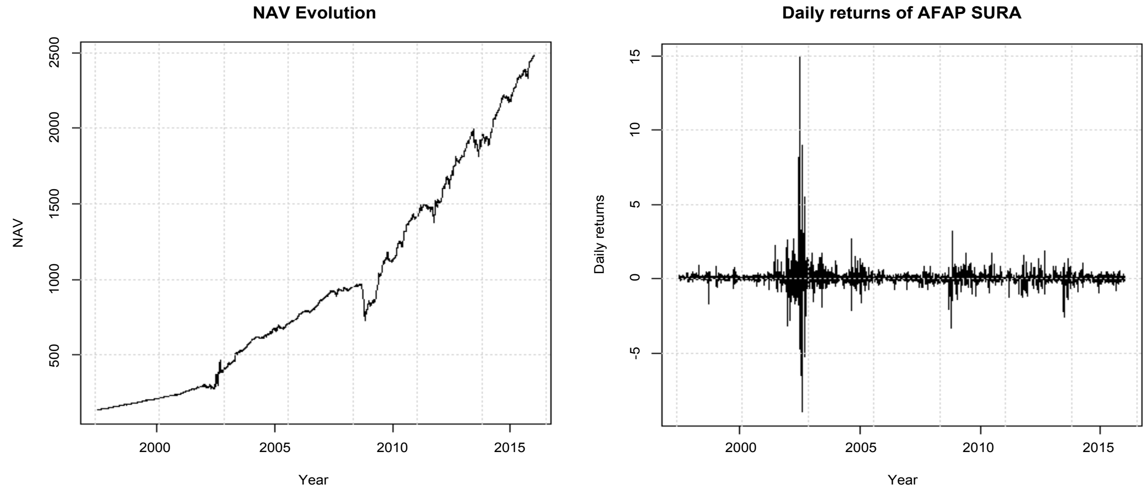


Table 1 shows the summary statistics for the series of daily changes. This table shows that kurtosis value is 193.33 and skewness value is 5.08. Relative value of Normal distribution is 3 and 0, respectively. Then there is no compatibility between the empirical distribution of daily returns and anormal distribution.

Table 1: Summary statistics for daily returns

Min	1st quarter	median	3rd quarter	max
-8,95	-0,04	0,05	0,16	14,88
Mean	sd	variance	skewness	kurtosis
0,06	0,53	0,28	5,08	193,33

The Jarqua-Bera statistic shows that the behavior of daily losses is different from normal distribution. The JB test statistics is defined as (Jarque and Bera, 1980):

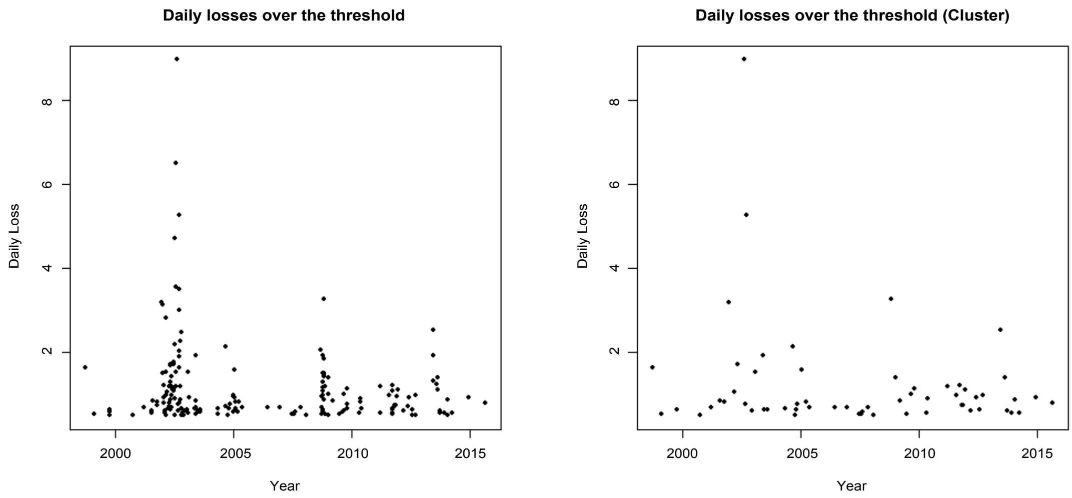
$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(C - 3)^2\right)$$

Where n is the number of observations, S is the sample skewness and C is the sample kurtosis. The JB statistic has approximately a chi-squared distribution, with two degrees of freedom. The Jarqua-Bera test depends on skewness and kurtosis statistics. If the JB test statistic equals zero, it means that the distribution has zero skewness and kurtosis is about equal 3, and so it can be concluded that the normality assumption holds.

Skewness values far from zero and kurtosis values far from 3 lead to an increase in JB values. The test returns the logical value $h = 1$ if it rejects the null hypothesis at the $p < 0.05$ significance level, and $h = 0$ otherwise. We found that JB value equals *7,505,400*, $p \sim 0$, $h = 1$, which implies that we reject the hypothesis of normality.

In practice, we have to consider two important aspects, the selection of the threshold u and the independence of the exceedances, that is, the independence of values that are above the threshold. For example, the left panel of Figure 5 shows 182 exceedances for the threshold $u = 0.5$, clearly there is a concentration of exceedances in the years 2002 and 2009. In the right panel we use a cluster technique to reduce dependence of the exceedances and we identify 59 exceedances. The clusters are identified as follows. The first exceedance of the threshold initiates the first cluster. The first cluster then remains active until either ten consecutive values fall below (or are equal to) the threshold. The next exceedance of the threshold (if it exists) initiates the second cluster, and so on. Thanks to this cluster technique we obtain exceedances that are independent as appear in the right panel of Figure 5.

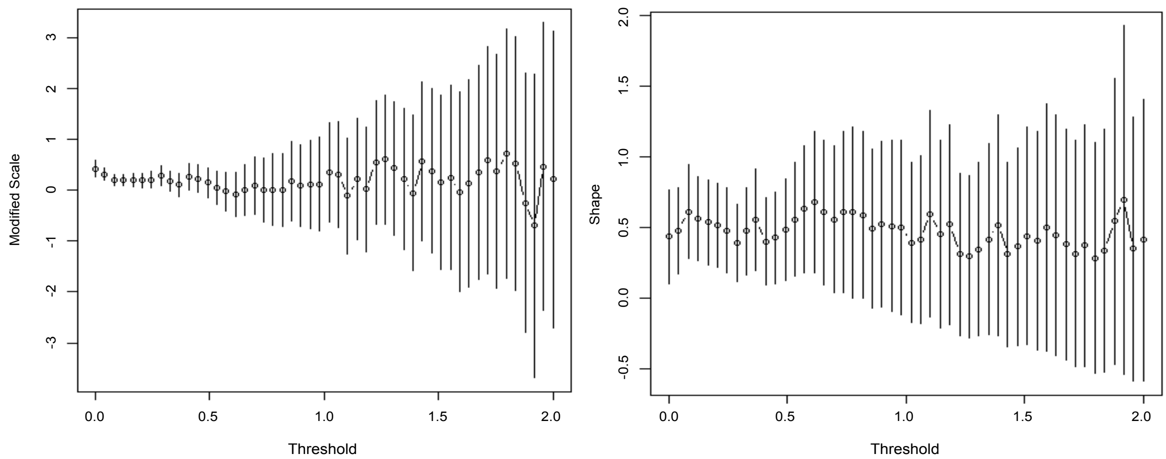
Figure 5: Daily losses over the threshold $u = 0.5$.



The choice of the threshold u is important, if an excessively high u results in too few exceedances and consequently high variance estimators. On the other hand, a too small u biases the estimators and the approximation to a GPD is not feasible (Embrechts, 1999). So far, there is no algorithm with a satisfactory performance for the selection of the threshold u available (Gilli and Kellezi, 2006). The issue of determining the fraction of data belonging to the tail is treated in Danielsson and de Vries (1997), Danielsson et al. (2001) and Dupuis (1998). However these references do not provide a clear answer to the question of which method should be used. For this reason the choice of u is a trade-off between bias and variance, for which there are no general guidelines. We use common-sense judgement and graphical approaches to select the threshold u.

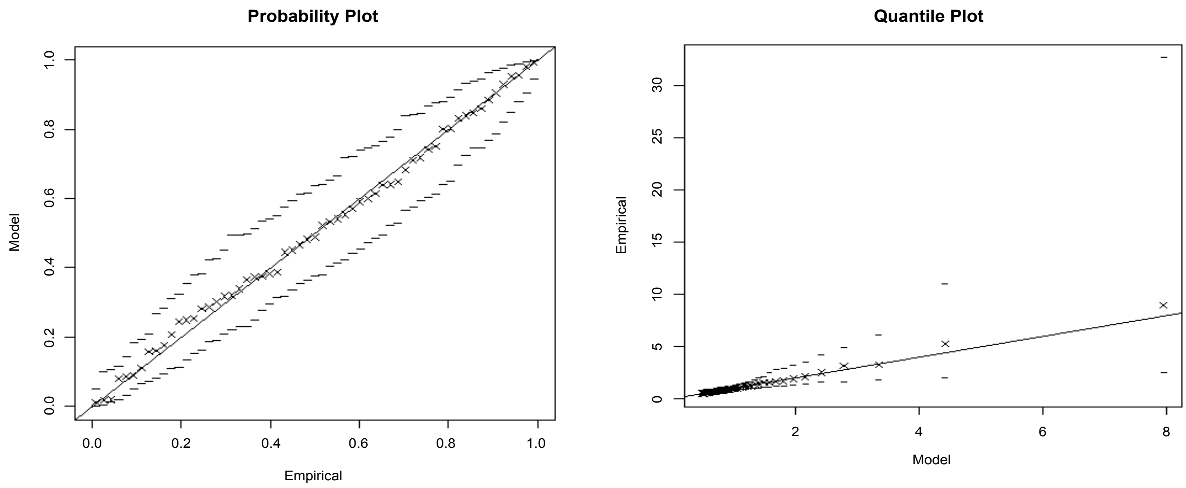
For different thresholds u, the maximum likelihood estimates for the shape and the modified scale parameter (modified by subtracting the shape multiplied by the threshold) are plotted against the thresholds (see Figure 6). If the threshold u is a valid threshold to be used for peaks over threshold modeling, the parameter estimates depicted should be approximately constant above u. Based on Figure 6, we choose the threshold $u = 0.5$ because the parameter estimates are approximately constant above 0.5.

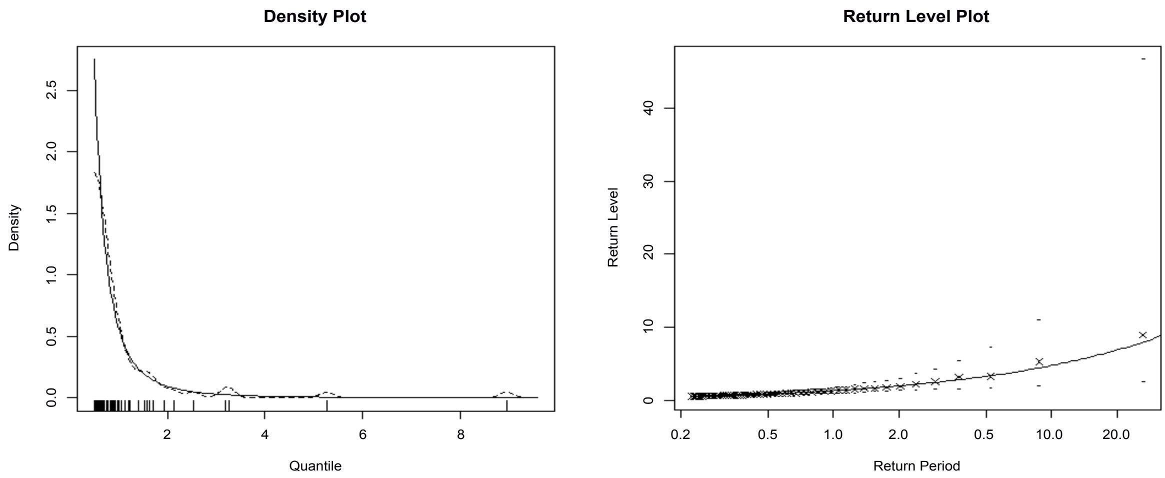
Figure 6: Estimates for the shape and the modified scale parameter for different thresholds *u*.



The results of maximum likelihood estimation of the GPD parameters (with the chosen threshold $u = 0.5$) are $\xi = 0.5175$ *(s.e 0.1919)* and $\beta = 0.3568$ *(s.e 0.0792)*. Figure 7 shows how GPD fits to the 59 exceedances.

Figure 7: Diagnostics plot for GPD model.



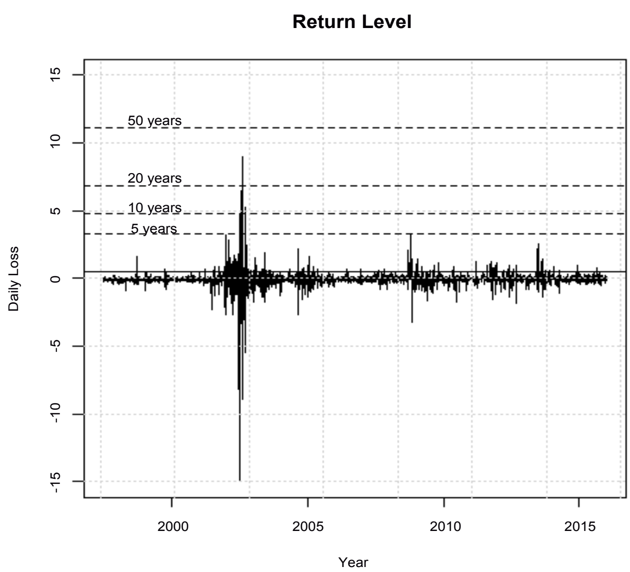


One of the purposes of this paper is to determine the maximum daily loss of the portfolio. In Table 2 we show the return level for different periods of time. The return levels are interpreted as follows, a maximum daily loss of 6.88% in the portfolio is expected once every twenty years. These estimates are consistent with the empirical return observed in Figure 8. The level of return can be interpreted as a stress loss of the portfolio, it is for this reason that it is important for workers to have a notion of the risk assumed by the pension funds.

Table 2: Return level for different periods of time.

	5 years	10 years	20 years	50 years
Return level	3,26%	4,75%	6,88%	11,14%

Figure 8: Return Level.



In Tables 3 and 4 we report 95%, 97.5%, 99% and 99.5% Value at Risk and Expected Shortfall estimates for two different estimation methods. The performance of the methods can be evaluated by comparing the estimates with the actual losses observed. VaR approaches based on the assumption of normal distribution overestimate low percentiles (due to the high variance estimation), and underestimate high percentiles (due to heavy tails). The absence of extreme values in the assumption of normal distribution underestimates the Expected Shortfall estimation for high percentiles. In turn, the extreme value approach on GPD models appears consistent with the actual losses observed as show the mean square error (MSE).

Table 3: Value at Risk: one day horizon estimates for two different estimation methods

	$\alpha = 5\%$	$\alpha =2.5\%$	$\alpha =1\%$	$\alpha = 0.5\%$	MSE
Normal model	$0.808_{(0.411)}$	$0.975_{(0.311)}$	$1.169_{(-0.032)}$	$1.301_{(-0.468)}$	0.121
GPD model	$0.408_{(0.011)}$	$0.666_{(0.002)}$	$1.185_{(-0.016)}$	$1.777_{(0.008)}$	0.000
Empirical Result	0.397	0.664	1.201	1.769	

Table 4: Expected Shortfall: one day horizon estimates for two different estimation methods

	$\alpha = 5\%$	$\alpha =2.5\%$	$\alpha =1\%$	$\alpha = 0.5\%$	MSE
Normal model	$1.030_{(0.039)}$	$1.175_{(-0.293)}$	$1.348_{(-0.966)}$	$1.468_{(-1.666)}$	0.949
GPD model	$1.049_{(0.058)}$	$1.583_{(0.115)}$	$2.658_{(0.344)}$	$3.887_{(0.753)}$	0.175
Empirical Result	0.991	1.468	2.314	3.134	

5. Discussion

In recent years volatility of international financial system has become severe and, consequently, risk management in Uruguayan pension funds has received extensive attention. As a measurement of market risk, VaR has been widely used in risk management. Uruguayan pension funds are exposed to this volatility, then we argue the need to communicate the risk they assume, not just profitability results. We understand that this will improve the transparency of Uruguay's pension system and allow members to have all the information about the management of their pension fund.

Traditional statistical methods for financial risk measures assume normal distribution for financial returns even when empiric distribution is not normal, which always causes errors in the estimation. Aiming at this problem, we utilized alternative approaches based in the Extreme Value Theory. The distinguish features of an extreme value analysis as the objective to quantify the stochastic behavior of a process at unusually large levels. In particular, extreme value analyses usually require estimation of the probability of events that are more extreme than any that have already been observed.

We have illustrated how Extreme Value Theory can be used to model financial risk measures such as Value at Risk, Expected Shortfall and Return Level, applying it to daily returns of AFAP SURA. Our major conclusion is that the POT model can be useful for assessing the size of extreme events. From a practical point of view we discussed how to handle the selection of the threshold u and the independence of the exceedances. After that we estimate the model parameters through maximum likelihood and quantified the return level for 5, 10, 20 and 50 years. Next, we compared traditional methods for risk measures with the POT model, noting that the last one provides a superior adjustment. This is because traditional models do not take into account the instability of financial markets that cause extreme values.

A possible extension of this research is raised by Singh et al. (2011), who propose a dynamic VaR forecasting method using EVT and GARCH regressions to model market volatility. GARCH models to forecast the estimates of conditional volatility provide dynamics of one day ahead forecasts for VaR and ES for the financial time series.

Finally, we invite the readers to continue deepening in the Theory of the Extreme Value and its applications in different areas of the science as, ocean wave modeling (Dawson, 2000); memory cell failure (McNulty et al., 2000); wind engineering (Harris, 2001); management strategy (Dahan & Mendelson, 2001); biomedical data processing (Roberts, 2000); thermodynamics of earthquakes (Lavenda & Cipollone, 2000); assessment of meteorological change (Thompson et al., 2001); non-linear beam vibrations (Dunne & Ghanbari, 2001); and food science (Kawas & Moreira, 2001).

References

Artzner, P.; Delbaen, F.; Eber, J. M.; Heath, D. (1999). Coherent Measures of Risk. Mathematical Finance 9 (3): 203.

Balkema, A., and de Haan, L. (1974). Residual life time at great age, Annals of Probability, 2, 792-804.

Coles, S. (2001). An Introduction to Statistical Modeling of Extreme Values. Springer-Verlag, London.

Danielsson, J. and de Vries, C. (1997). Beyond the Sample: Extreme Quantile and Probability Estimation. Mimeo.

Danielsson, J., de Vries, C., de Haan, L., and Peng, L. (2001). Using a Bootstrap Method to Choose the Sample Fraction in Tail Index Estimation. Journal of Multivariate Analysis, 76(2):226-248.

Dupuis, D. J. (1998). Exceedances over high thresholds: A guide to threshold selection. Extremes, 1(3):251-261.

Dowd, K. (2002) Measuring Market Risk. John Wiley & Sons.

Embrechts, P. (1999). Extreme Value Theory as a Risk Management Tool. North American Actuarial Journal, 3(2).

Embrechts P., Kluppelberg C. and Mikosch T. (1997) Modelling extremal events: for insurance and finance, Springer, Berlin.

Fisher, R.A.; Tippett, L.H.C. (1928). Limiting forms of the frequency distribution of the largest and smallest member of a sample, Proc. Cambridge Phil. Soc. 24: 180-190.

Forteza, A., Bucheli, M., Caristo, A, Siandra, E. (1999). La reforma de la seguridad social en Uruguay: efectos macroeconómicos y mercados de capitales (No. 368.4 FORr).

Gilli, M., Kellezi, E. (2006). An application of extreme value theory for measuring financial risk. Computational Economics 27, 207-228.

Gnedenko, B. V. (1948). On a local limit theorem of the theory of probability, Uspekhi Mat. Nauk, 3:3(25), 187-194.

Jarque, C. M; Bera, A. K. (1980). Efficient tests for normality, homoscedasticity and serial independence of regression residuals Economics Letters Volume 6, Issue 3, Pages 255-259.

Makarov, M. (2007) Applications of exact extreme value theorem. Journal of Operational Risk, Volume 2, number 1 pages 115-120.

McNeil A.J., Frey R. and Embrechts P. (2005) Quantitative risk management: Concepts, techniques and tools. Princeton University Press.

Pickands, J. (1975). Statistical inference using extreme order statistics, Annals of Statistics, 3, 119-131.

Ramaswamy, S (2004), Managing Credit Risk in Corporate Bond Portfolios: A Practitioner's Guide. John Wiley & Sons.

Reiss, R.-D. and Thomas, M. (2007). Statistical Analysis of Extreme Values with Applications to Insurance, Finance, Hydrology and Other Fields, Third Edition. Birkhauser, Basel.

Resnick, S. I. (1987) Extreme Values, Regular Variation, and Point Processes, Springer.

Rockafellar, R.T.; Uryasev S. (2002). Conditional value-at-risk for general loss distributions. Journal of Banking & Finance, Volume 26, Issue 7, Pages 1443-1471.

Singh, A. K, Allen, D., Powell D (2011). Value at Risk Estimation Using Extreme Value Theory. 19th International Congress on Modelling and Simulation. Perth, Australia, 12-16 December.

Tsay, R. S. (2007) Extreme values and their applications in finance.